The WSS scalar complex valued random process \( (X(t), t \in \mathbb{R}) \) is amplitude modulated onto a carrier to produce the signal
\[
Z(t) = \sqrt{2} X(t) \cos(2\pi f_c t + \Theta),
\]
where \( \Theta \) is uniformly distributed on \([0; 2\pi]\) and is independent of the process \( (X(t), t \in \mathbb{R}) \).

(a) Demonstrate that \( (Z(t), t \in \mathbb{R}) \) is WSS.

(b) Derive an expression for the autocorrelation function \( R_{ZZ}(\tau) \) in terms of the autocorrelation function \( R_{XX}(\tau) \).

(c) Derive an expression of the power spectral density \( S_{ZZ}(\omega) \) in terms of \( S_{XX}(\omega) \). Give a physical interpretation for the derived formula.

Suppose now to observe the process \( (Y(t), t \in \mathbb{R}) \) given by
\[
Y(t) = \alpha Z(t) + \beta Z(t - T) + W(t),
\]
where \( \alpha, \beta, \) and \( T \) are real-valued constants, and \( (W(t), t \in \mathbb{R}) \) is a white Gaussian noise with power spectral density \( N_0 \) and is independent of the process \( (Z(t), t \in \mathbb{R}) \).

(d) Derive an expression for the power spectral density \( S_{YY}(\omega) \) in terms of \( S_{ZZ}(\omega) \).

(e) What is the frequency response \( H(\omega) \) of the linear filter whose output \( (\hat{Z}(t), t \in \mathbb{R}) \) is the LLSE of \( (Z(t), t \in \mathbb{R}) \)?