UCSD ECE 35 Prerequisite Test Solutions

1. The product of a 2x2 matrix with a 2x1 vector is defined as

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{bmatrix}$$

We can use this to write the system of equations

$$y_1 = ax_1 + bx_2$$
$$y_2 = cx_1 + dx_2$$

as a matrix equation:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Based on this, we can express our system of equations as

$$\begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

To solve for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ we need to multiply both sides of this equation by the inverse of $\begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}$. The inverse of a 2x2 matrix is given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

So

$$\begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}^{-1} = \frac{1}{1(4) - 4(4)} \begin{bmatrix} 4 & -4 \\ -4 & 1 \end{bmatrix} = \frac{-1}{12} \begin{bmatrix} 4 & -4 \\ -4 & 1 \end{bmatrix}$$

Now we find $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \frac{-1}{12} \begin{bmatrix} 4 & -4 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ .25 \end{bmatrix}$$

So we find $x_1 = 1$ and $x_2 = .25$

2. We begin by adding the first equation to the second equation:

$$\begin{aligned} -3x_1 - 2x_2 + 7x_3 &= 5 \\ &+ \\ 3x_1 + 3x_2 - 4x_3 &= 7 \\ \hline x_2 + 3x_3 &= 12 \end{aligned}$$

Next, we solve this equation for x_2.

$$x_2 = 12 - 3x_3$$

Now, we substitute this equation into the third equation.

$$\begin{aligned} 4x_1 + 2(12 - 3x_3) + 6x_3 &= 10 \\ 4x_1 + 24 - 6x_3 + 6x_3 &= 20 \\ 4x_1 &= -4 \end{aligned}$$

Thus

$$x_1 = -1$$

Substituting this value for x_1 into the first equation gives

$$\begin{aligned} 3 - 2x_2 + 7x_3 &= 5 \\ -2x_2 + 7x_3 &= 2 \end{aligned}$$

Solving this for x_2 gives

$$x_2 = \frac{7}{2}x_3 - 1$$

Now we plug in this equation for x_2 and $x_1 = 1$ into the second equation and solve for x_3:

$$\begin{aligned} -3 + 3(\frac{7}{2}x_3 - 1) - 4x_3 &= 7 \\ \frac{21}{2}x_3 - 4x_3 &= 13 \\ \frac{13}{2}x_3 &= 13 \\ x_3 &= 2 \end{aligned}$$

Lastly, we plug $x_3 = 2$ into $x_2 = \frac{7}{2}x_3 - 1$:

$$\begin{aligned} x_2 &= \frac{7}{2} * 2 - 1 \\ x_2 &= 6 \end{aligned}$$

Therefore $x_1 = -1$, $x_2 = 6$, and $x_3 = 2$.

Alternatively, we can solve this problem using the matrix approach developed in exercise 2:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & 7 \\ 3 & 3 & -4 \\ 4 & 2 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 7 \\ 20 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 2 \end{bmatrix}$$

The downside to this approach is that calculating the inverse of a 3x3 matrix is often quite difficult. However, in situations when a calculator is allowed, this approach is often much simpler than solving a system by hand.

3. The integral of $y(t)$ from $-\infty$ to ∞ is the total area under the curve. Since $y(t) = 0$ outside of [-1,1], the integral is just the area of the triangle.

$$\int_{-\infty}^{\infty} y(t)dt = \frac{1}{2} * 2 * 1 = 1$$

4. (a)

$$\frac{df(t)}{dt} = 2at + b$$

(b)

$$\int_{t_1}^{t_2} f(t)dt = \frac{a}{3}t^3 + \frac{b}{2}t^2 + ct\Big|_{t_1}^{t_2}$$

$$= \frac{a}{3}t_2^3 + \frac{b}{2}t_2^2 + ct_2 - \frac{a}{3}t_1^3 - \frac{b}{2}t_1^2 - ct_1$$

$$= \frac{a}{3}(t_2^3 - t_1^3) + \frac{b}{2}(t_2^2 - t_1^2) + c(t_2 - t_1)$$

5. (a) To find $I(t)$, we plug $\frac{dV(t)}{dt}$ into the equation.

$$\frac{dV(t)}{dt} = \omega * cos(\omega t)$$

Therefore

$$I(t) = C * \omega * cos(\omega t)$$

(b) To find $I(t)$, we plug $V(t)$ into the equation.

$$\frac{cos(\omega t)}{L} = \frac{dI(t)}{dt}$$

Integrating both sides with respect to t we find

$$I(t) = \frac{1}{L}\int cos(\omega t)dt = \frac{1}{L * \omega} sin(\omega t) + c$$

6. A complex number z can be written in rectangular form as $z = a + jb$ or in phasor form as $z = |z|\angle\theta$. To convert between these different forms, we have the relationships:

$$a = |z|\cos\theta$$
$$b = |z|\sin\theta$$

and

$$|z| = \sqrt{a^2 + b^2}$$
$$tan(\theta) = \frac{b}{a}$$

So we have:

(a) $|z| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ and $\theta = 45°$
(b) $|z| = \sqrt{3^2 + 0^2} = 3$ and $\theta = 0°$
(c) $|z| = \sqrt{0^2 + (-2)^2} = 2$ and $\theta = 270°$
(d) $|z| = \sqrt{(-12)^2 + 3^2} = 3\sqrt{17}$ and $\theta = 165.96°$

7. It is usually easier to add/subtract complex numbers in rectangular form and to multiply/divide them in phasor form. For complex numbers $z_1 = a + jb = |z_1|\angle\theta_1$ and $z_2 = c + jd = |z_2|\angle\theta_2$, we have:

$$z_1 + z_2 = a + jb + c + jd = (a + c) + j(b + d)$$
$$z_1 - z_2 = a + jb - (c + jd) = (a - c) + j(b - d)$$
$$z_1 * z_2 = |z_1||z_2|\angle(\theta_1 + \theta_2)$$
$$\frac{z_1}{z_1} = \frac{|z_1|}{|z_2|}\angle(\theta_1 - \theta_2)$$

Also, for a complex number $z = a + jb = |z|\angle\theta$, the complex conjugate z^* is given by

$$z^* = a - jb = |z|\angle{-\theta}$$

So we have:

(a) $(4 + 3j) - (2 - 6j) = 2 + 9j = \sqrt{85}\angle 77.47°$
(b) $(1 + 2j)(4 + 6j) = (\sqrt{5}\angle 63.4°)(2\sqrt{13}\angle 56.3°) = 2\sqrt{65}\angle 119.7°$
(c) $(1 + 2j)(4 - 6j) = (\sqrt{5}\angle 63.4°)(2\sqrt{13}\angle - 56.3°) = 2\sqrt{65}\angle 7.1°$
(d) $\frac{(2 + 4j)}{(6 - 7j)} = \frac{2\sqrt{5}\angle 63.43°}{\sqrt{85}\angle - 49.4°} = \frac{2}{\sqrt{17}}\angle 112.83°$
(e) $\frac{(1 + 2j) + (3 + 4j)}{(2 - 3j) - 4} = \frac{4 + 6j}{-2 - 3j} = \frac{2(2 + 3j)}{-1(2 + 3j)} = -2 = 2\angle 180°$

(f) $((1+2j)(2+3j))^* = ((\sqrt{5}\angle 63.4°)(\sqrt{13}\angle 56.3°))^* = (\sqrt{65}\angle 119.7°)^*$
$= \sqrt{65}\angle -119.7°$

8. (a) $\int cos(t)dt = sin(t) + c$

   (b) $\int_0^t cos(t)dt = sin(t) - sin(0) = sin(t)$

   (c) $\int \frac{5}{x}dx = 5\int \frac{1}{x} = 5ln(|x|) + c$

   (d) $\int e^x dx = e^x + c$

   (e) $\int e^{jx}dx = \dfrac{e^{jx}}{j} + c = -je^{jx} + c$

   (f) For this integral, we need to first notice that the answer is a function of t. Also,

   $$x(\tau)e^{j\tau} = \begin{cases} e^{j\tau} & -3 < \tau < 3 \\ 0 & else \end{cases}$$

   So from $-\infty < t < -3$, we have $\int_{-\infty}^t 0 d\tau = 0$

   From $-3 < t < 3$ we have $\int_{-\infty}^t x(\tau)e^{j\tau}d\tau = \int_{-3}^t e^{j\tau}d\tau = \frac{e^{jt}-e^{-j3}}{j} = -j(e^{jt} - e^{-j3})$

   Lastly from $3 < t < \infty$ we have $\int_{-\infty}^t x(\tau)e^{j\tau}d\tau = \int_{-3}^3 e^{j\tau}d\tau = -j(e^{3j} - e^{-3j})$

   Therefore our final solution is $\int_{-\infty}^t x(\tau)e^{j\tau}d\tau = \begin{cases} 0 & -\infty < t < -3 \\ -j(e^{jt} - e^{-j3}) & -3 < t < 3 \\ -j(e^{3j} - e^{-3j}) & 3 < t < \infty \end{cases}$

   (g) $\int_0^y xe^{-x^2}dx = \dfrac{-e^{-x^2}}{2}|_0^y = \dfrac{-e^{-y^2}}{2} - \dfrac{-1}{2} = \dfrac{1-e^{-y^2}}{2}$

   (h) $\int_{-\frac{y}{2}}^{\frac{y}{2}}(2x+4)dx = x^2 + 4x|_{\frac{-y}{2}}^{\frac{y}{2}} = \frac{y^2}{4} + 2y - \frac{y^2}{4} - (-2y) = 4y$