MS/CTS EXAM – FALL 2014

In a digital communication system data is transmitted via a pulse train appearing as

$$X(t) = \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_0)$$

where $g(t)$ is a real, non-random function (pulse) and T_0 is the spacing between adjacent pulses. The α_n s are independent, identically distributed random variables with zero mean, variance σ^2 and $g(t) = \exp[-|t|]$. In general, such pulse trains are not wide sense stationary; they are known as cyclostationary processes (that is, they satisfy $R_X(t+T_0, s+T_0) = R_X(t,s)$). Evaluate $P(\omega)$, the power spectral density of X(t). [HINT: Begin by showing that, for a function $f(t)$ which is periodic with period T_0 and integrable over one period,

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)dt = \frac{1}{T_0} \int_{0}^{T_0} f(t)dt \,,$$

then use this property to evaluate the power spectral density.]

$P(\omega) =$

FOURIER TRANSFORM PAIRS

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega},\ a>0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega},\ a>0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a\|t\|} \leftrightarrow \frac{2a}{a^2+\omega^2},\ a>0$	16	$\delta(t-t_o) \leftrightarrow e^{-j\omega t_o}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a\|\omega\|},\ a>0$	17	$e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega-\omega_o)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2},\ a>0$	18	$\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}},\ a>0$	19	$\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
7	$\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau\,\text{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)] + \frac{j\omega}{\omega_o^2-\omega^2}$
8	$\text{sinc}(Wt) \leftrightarrow \frac{\pi}{W}\text{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)] + \frac{\omega_o}{\omega_o^2-\omega^2}$
9	$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2}\text{sinc}^2(\frac{\omega\tau}{4})$	22	$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W}\Delta(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$
11	$e^{-at}\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2+\omega_o^2},\ a>0$	24	$\sum_{n=-\infty}^{\infty}\delta(t-nT) \leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\frac{2\pi}{T})$
12	$e^{-at}\cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2+\omega_o^2},\ a>0$	25	$\sum_{n=-\infty}^{\infty} f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty}\frac{1}{T}F(\omega-n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$		

$$\text{sinc}(t) \equiv \frac{\sin(t)}{t}$$