CTS MS EXAM SOLUTION – SPRING 2015

With a rate λ $\quad P(N(\tau)=k) = \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau}$

$k = 0, 1, \ldots$

then

$$F_{T_M}(\tau) = P(T_M \le \tau) = \sum_{k=M}^{\infty} P(N(\tau)=k)$$

$$= \sum_{k=M}^{\infty} \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau}$$

$$f_{T_M}(\tau) = \frac{d}{d\tau} F_{T_M}(\tau) = \sum_{k=M}^{\infty} k \frac{\lambda^k}{k!} \tau^{k-1} e^{-\lambda\tau}$$

$l = k-1$

$$- \lambda \sum_{k=M}^{\infty} \frac{\lambda^k \tau^k}{k!} e^{-\lambda\tau}$$

$l = k$

$$= \lambda \sum_{l=M-1}^{\infty} \frac{(\lambda\tau)^l}{l!} e^{-\lambda\tau}$$

$$- \lambda \sum_{l=M}^{\infty} \frac{(\lambda\tau)^l}{l!} e^{-\lambda\tau}$$

$$= \frac{(\lambda\tau)^{M-1}}{(M-1)!} \lambda e^{-\lambda\tau}$$

$$\therefore \boxed{f_{T_M}(\tau) = \frac{(\lambda\tau)^{M-1}}{(M-1)!} \lambda e^{-\lambda\tau}}$$

$$P_e = P(T_M \geq T_0 \mid \lambda = \lambda_2) \cdot P(\lambda = \lambda_2)$$

$$+ P(T_M < T_0 \mid \lambda = \lambda_1) \cdot P(\lambda = \lambda_1)$$

$$\boxed{P_e = \frac{1}{2} \int_{T_0}^{\infty} \frac{(\lambda_2 \tau)^{M-1}}{(M-1)!} \lambda_2 e^{-\lambda_2 \tau} \, d\tau + \frac{1}{2} \int_0^{T_0} \frac{(\lambda_1 \tau)^{M-1}}{(M-1)!} \lambda_1 e^{-\lambda_1 \tau} \, d\tau}$$

choose T_0 to minimize P_e; that is

$$0 \stackrel{set}{=} \frac{d}{dT_0} P_e = \frac{-1}{2} \frac{(\lambda_2 T_0)^{M-1}}{(M-1)!} \lambda_2 e^{-\lambda_2 T_0}$$

$$+ \frac{1}{2} \frac{(\lambda_1 T_0)^{M-1}}{(M-1)!} \lambda_1 e^{-\lambda_1 T_0}$$

$$\frac{(\lambda_2 T_0)^{M-1}}{(M-1)!} \lambda_2 e^{-\lambda_2 T_0} = \frac{(\lambda_1 T_0)^{M-1}}{(M-1)!} \lambda_1 e^{-\lambda_1 T_0}$$

$$\lambda_2^M e^{-(\lambda_2 T_0)} = \lambda_1^M e^{-(\lambda_1 T_0)}$$

$$\therefore \quad \boxed{T_0 = M \frac{\ln(\lambda_2) - \ln(\lambda_1)}{\lambda_2 - \lambda_1}}$$