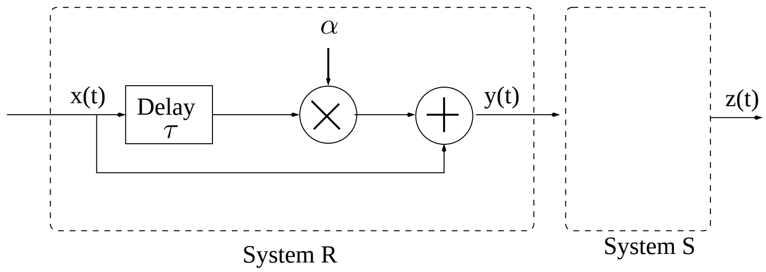
Signals and Systems problem for the Spring 2014 MS Exam in ECE

Suppose two continuous-time systems R and S are cascaded as shown in the diagram below. The system R has input $x(t)$ and output $y(t)$, and system S has input $y(t)$ and output $z(t)$. The assumptions in part (a) are independent from those of part (b), so do not mix them up.



(a) For the special case of $\alpha = 1/2$ and $\tau = 250$ microseconds, suppose the system S is an ideal low-pass filter on the interval ± 1000 Hz. Find the magnitude of the Fourier transform of the signal $z(t)$ if $x(t)$ is a delta function. Carefully label the plot.

SOLUTION:

From the block diagram, the input/output relationship is $y(t) = x(t) + \alpha x(t-\tau)$. When $x(t) = \delta(t)$ we have $X(\omega) = 1$. Taking Fourier transforms gives

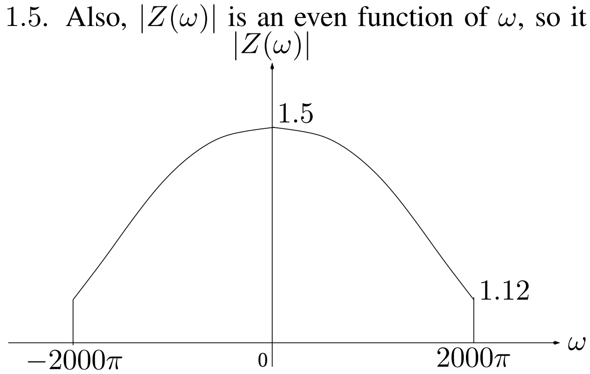
$$H_R(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 + \alpha e^{-j\omega\tau}$$

$$Z(\omega) = X(\omega) \cdot \underbrace{\frac{Y(\omega)}{X(\omega)}}_{H_R(\omega)} \cdot \underbrace{\frac{Z(\omega)}{Y(\omega)}}_{H_S(\omega)}$$

$$\therefore |Z(\omega)| = |H_S(\omega)| \cdot \sqrt{(1+\alpha\cos(\omega\tau))^2 + \alpha^2 \sin^2(\omega\tau)}$$

$$= \begin{cases} \sqrt{(1+\alpha\cos(\omega\tau))^2 + \alpha^2 \sin^2(\omega\tau)} & \text{if } |\omega| \le 2000\pi \\ 0 & \text{else} \end{cases}$$

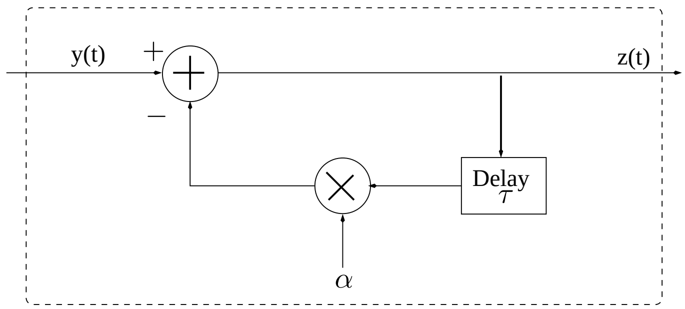
When $f = 1000$ Hz we have $\omega = 2\pi f = 2000\pi$, so $\omega\tau = (2000\pi)(250 \cdot 10^{-6}) = \pi/2$, $\cos(\omega\tau) = 0$, and $\sin(\omega\tau) = 1$, which implies $|Z(\omega)| = \sqrt{1+\alpha^2} = \sqrt{5}/2 \approx 2.24/2 = 1.12$. When $\omega = 0$, we have $|Z(\omega)| = 1 + \alpha = 1.5$. Also, $|Z(\omega)|$ is an even function of ω, so it is symmetrical about $\omega = 0$.



(b) Suppose the system S is such that $z(t) = x(t)$. Draw a block diagram of system S Carefully label your blocks and signals.

SOLUTION:

From the block diagram, the input/output relationship is $y(t) = x(t) + \alpha x(t - \tau)$. Solving for $x(t)$ gives $x(t) = y(t) - \alpha x(t - \tau)$. This can be implemented in the block diagram shown below, where we changed notation $x \longrightarrow z$:



System S