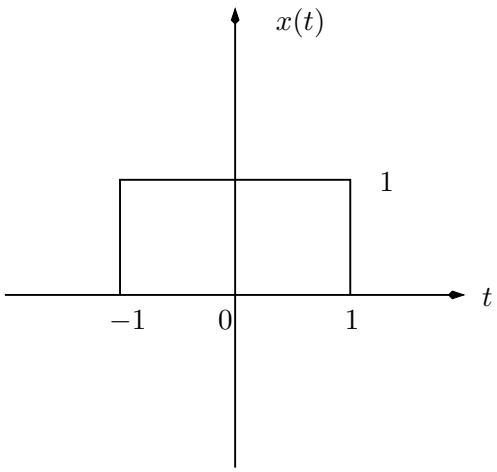
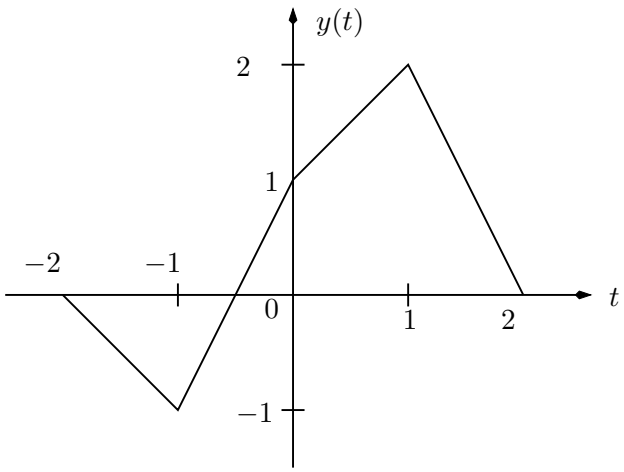
Signals and Systems problem for the Fall 2014 MS Exam in ECE

Suppose the time function $x(t)$ shown below is input to a linear time-invariant system and the output time function is $y(t)$ shown below. Find the impulse response of the system.





SOLUTION: Let $x_1(t) = x'(t)$, $y_1(t) = y'(t)$, and $y_2(t) = y''(t)$.
Use Laplace transforms (and let $u = e^{-s}$):

$$x_1(t) = \delta(t+1) - \delta(t-1)$$
$$y_2(t) = -\delta(t+2) + 3\delta(t+1) - \delta(t) - 3\delta(t-1) + 2\delta(t-2)$$
$$X_1(s) = e^s - e^{-s} = \frac{(u-1)(u+1)}{-u}$$
$$Y_2(s) = -e^{2s} + 3e^s - 1 - 3e^{-s} + 2e^{-2s} = \frac{2u^4 - 3u^3 - u^2 + 3u - 1}{u^2} = \frac{(u-1)^2(u+1)(2u-1)}{u^2}$$
$$y(t) = x(t) * h(t)$$
$$y''(t) = x'(t) * h'(t)$$
$$H_1(s) = \frac{Y_2(s)}{X_1(s)} = \frac{(u-1)(2u-1)}{-u} = -2u + 3 - u^{-1} = -2e^{-s} + 3 - e^s$$
$$h_1(t) = -2\delta(t-1) + 3\delta(t) - \delta(t+1)$$
$$h(t) = \int_{-\infty}^{t} h_1(\tau)d\tau = \begin{cases} -1 & \text{if } -1 \le t < 0 \\ 2 & \text{if } 0 \le t < 1 \\ 0 & \text{else} \end{cases}$$