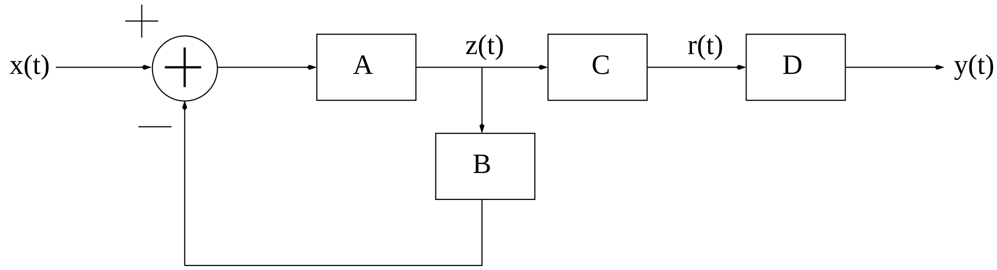
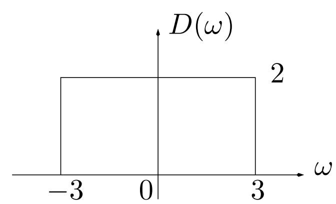
Signals and Systems problem for the Spring 2015 MS Exam in ECE

In the following block diagram, linear time-invariant systems A, B, C, and D have impulse responses $e^{-3t}u(t)$, $e^{-t}u(t)$, $e^{-t}u(t)$, and $(2/\pi)t^{-1}\sin(3t)$, respectively. Let $h_1(t)$, $h_2(t)$, and $h_3(t)$ denote the outputs $z(t)$, $r(t)$, and $y(t)$, respectively, when the input $x(t)$ is an impulse (i.e. Dirac delta function). Determine $h_2(t)$ and the magnitude of the Fourier Transform of $h_3(t)$, and plot each of them.



SOLUTION:
We have $A(\omega) = \frac{1}{j\omega+3}$, $B(\omega) = C(\omega) = \frac{1}{j\omega+1}$, and $D(\omega)$ is the low pass filter shown in the diagram.



We have:

$$A(\omega)(X(\omega) - B(\omega)Z(\omega)) = Z(\omega)$$

$$H_1(\omega) = \frac{Z(\omega)}{X(\omega)} = \frac{A(\omega)}{1 + A(\omega)B(\omega)} = \frac{\frac{1}{j\omega+3}}{1 + \frac{1}{j\omega+3} \cdot \frac{1}{j\omega+1}} = \frac{j\omega + 1}{(j\omega + 2)^2}$$

$$H_2(\omega) = \frac{R(\omega)}{X(\omega)} = \frac{R(\omega)}{Z(\omega)} \cdot = \frac{Z(\omega)}{X(\omega)} = C(\omega)H_1(\omega) = \frac{1}{(j\omega + 2)^2}$$

$$\therefore h_2(t) = te^{-2t}u(t)$$

$$H_3(\omega) = \frac{R(\omega)}{X(\omega)} \cdot \frac{Y(\omega)}{R(\omega)} = H_2(\omega)D(\omega) = \frac{D(\omega)}{(j\omega + 2)^2}$$

$$\therefore |H_3(\omega)| = \frac{|D(\omega)|}{\omega^2 + 4} = \begin{cases} \frac{2}{\omega^2+4} & \text{if } |\omega| < 3 \\ 0 & \text{else} \end{cases}$$