Math Question (equal weight each part) - ECE MS Comp Exam, Fall 2013

Part 1

(i) Solve the following set of coupled differential equations

$$\begin{aligned} \frac{dA(z)}{dz} &= j\frac{\gamma}{2}B(z) \\ \frac{dB(z)}{dz} &= j\frac{\gamma}{2}A(z) \end{aligned}$$

subject to the boundary conditions that $A(0) = C$ and $B(0) = 0$. (Note that $j \doteq \sqrt{-1}$)

(ii) Sketch the solutions for $|A(z)|^2$ and $|B(z)|^2$ for a value of $\gamma = \pi/4$.

Part 2

A zero-mean joint Gaussian probability distribution is given by

$$f_{\underline{xy}}(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right).$$

(i) Derive the probability distribution $f_{\underline{z}}(z)$ for the random variable

$$\underline{z} \doteq \underline{x}^2 + \underline{y}^2,$$

which is the squared-magnitude of the joint Gaussian probability distribution and sketch this distribution for $\sigma^2 = 1$.

(ii) Derive the probability distribution $f_{\underline{\phi}}(\phi)$ for the random variable

$$\underline{\phi} \doteq \tan^{-1}\left(\underline{y}/\underline{x}\right),$$

which is the phase of the joint Gaussian distribution and sketch this distribution for $\sigma^2 = 1$.