

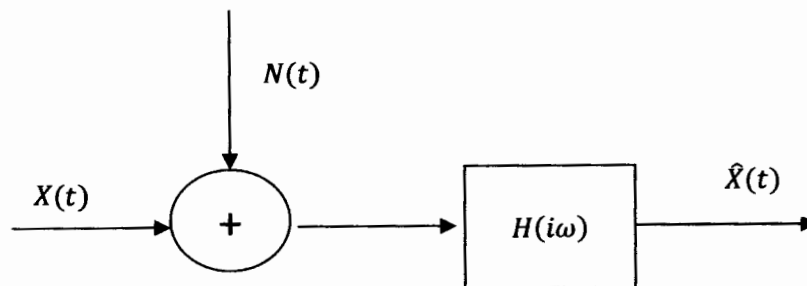
MS EXAM - CTS - FALL 2013

During transmission a random process, $X(t)$, is corrupted by additive noise, $N(t)$. These processes are independent, wide sense stationary and both have zero mean. Their respective correlation functions are:

$$R_X(\tau) = e^{-a|\tau|} \quad \text{and} \quad R_N(\tau) = e^{-b|\tau|}.$$

The sum, $X(t) + N(t)$, is passed through an ideal low-pass filter with one-sided bandwidth Ω . That is

$$H(i\omega) = \begin{cases} 1, & -\Omega \leq \omega \leq \Omega \\ 0, & \text{otherwise} \end{cases}.$$



Determine the bandwidth Ω_0 that minimizes the error $\mathcal{E} = E[(X(t) - \hat{X}(t))^2]$

$\Omega_0 =$

FOURIER TRANSFORM PAIRS

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega$$

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}, a > 0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}, a > 0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a t } \leftrightarrow \frac{2a}{a^2+\omega^2}, a > 0$	16	$\delta(t-t_0) \leftrightarrow e^{-j\omega t_0}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a \omega }, a > 0$	17	$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}, a > 0$	18	$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}, a > 0$	19	$\sin(\omega_0 t) \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
7	$\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau \text{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_0 t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
8	$\text{sinc}(Wt) \leftrightarrow \frac{\pi}{W} \text{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_0 t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
9	$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	22	$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W} \Delta(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$
11	$e^{-at} \sin(\omega_0 t)u(t) \leftrightarrow \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}, a > 0$	24	$\sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - n\frac{2\pi}{T})$
12	$e^{-at} \cos(\omega_0 t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2 + \omega_0^2}, a > 0$	25	$\sum_{n=-\infty}^{\infty} f(t)\delta(t - nT) \leftrightarrow \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\omega^2}{2}}$		

$$\text{sinc}(t) \equiv \frac{\sin(t)}{t}$$

