

CTS MS EXAM QUESTION, SPRING 2014

Consider the compound Poisson process

$$X(t) = \begin{cases} 0, & N(t) = 0 \\ \sum_{k=1}^{N(t)} A_k, & N(t) > 0 \end{cases}$$

Here $N(t)$ is a classical Poisson process (independent increments, constant rate λ and $N(0) = 0$) and the A_k s are independent and identically distributed with zero mean and variance σ^2 .

(1) Evaluate the mean and correlation function of $X(t)$.

It is desired to estimate $X(t)$ using one prior observation. The estimate is given by $\widehat{X}(t) = K \cdot X(t - t_1)$.

(2) Determine the value of K that minimizes $\mathcal{E} = E[(X(t) - \widehat{X}(t))^2]$.

An answer not supported by appropriate reasoning will not receive any credit.

$E[X(t)] =$

$E[X(t)X(s)] =$

$K =$