MS Exam ISRC Spring 2014

Let x be an n-vector observation of the unknown p-vector θ. $x = H\theta + w$ where w is "white" with unknown variance σ. The LSE of θ. is $\theta^{\wedge} = (H'H)^{-1}H'x$. The inverse is assumed to exist. We will estimate σ as follows:

$(\sigma^{\wedge})^2 = (1/n)(x - H\theta^{\wedge})'\,)(x - H\theta^{\wedge})$

Show that this estimate biased. Note that for a symmetric matrix A, $x'Ax = tr(Axx')$