

Math Question (equal weight each part)

Part 1

(i) Solve the following differential equation

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 29y(t) = 87$$

subject to the initial conditions $y(0) = 0$ and $y'(0) = 6$.

(ii) Sketch the solution over the range of $0 < t < 5$.

(iii) Assuming this solution describes the behavior of a RLC circuit, describe the nature of the response and label the on the sketch the physically significant parts of the solution.

Part 2

(i) Find the eigenvalues and eigenvectors of the following matrix

$$M = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

(ii) Find the matrix inverse M^{-1}