NDS MS Exam FA '11 Solution:

(a) It would be conductance quanta $G = G_0 = 2e^2/h$, assume there's only spin degeneracy and no band degeneracy at the CB bottom.

(b) Using particle-in-a-box model, at y- and z-direction:

$$\left.\begin{aligned} E_y &= \frac{\hbar^2 k_y^2}{2m^*}, \quad k_y = \frac{n\pi}{W}, \quad n = 1, 2, 3 \ldots \\ E_z &= \frac{\hbar^2 k_z^2}{2m^*}, \quad k_z = \frac{m\pi}{W}, \quad m = 1, 2, 3 \ldots \end{aligned}\right\}$$

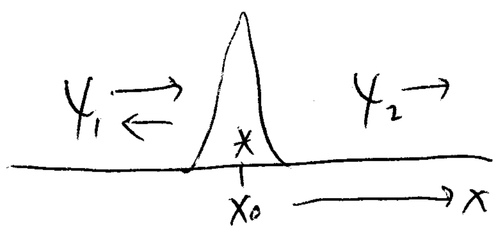
Total subband energy is $E = E_y + E_z$

Lowest subband is $(n, m) = (1, 1)$

2nd subband is $(n, m) = (1, 2)$ or $(2, 1)$ (degenerate).

So $$E_2 - E_1 = \delta E = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{W}\right)^2 \cdot \left[(1^2 + 2^2) - (1^2 + 1^2)\right] = \frac{3\pi^2\hbar^2}{2Wm^*}$$

(c)



Incoming wavefunction is $\psi_1 = e^{ikx} + B \cdot e^{-ikx}$

Transmitted wave function is $\psi_2 = C \cdot e^{ikx}$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$ is the wave vector

At the boundary x_0, $\psi_1(x_0^-) = \psi_2(x_0^+)$ are continuous. For simplificity let $x_0 = 0$.

$$\Rightarrow 1 + B = C \qquad (1)$$

Schrödinger's eq: $-\frac{\hbar^2}{2m}\psi'' + V\psi = E\psi$. (2)

At x_0, $V = V_0\lambda\,\delta(x - x_0) = V_0\lambda\,\delta(x)$ The only way to use this potential is to make use of $\int_{-\infty}^{\infty}\delta(x)dx = 1$.

Let's integrate (2) with $\int_{-\epsilon}^{+\epsilon} \cdots dx$, then let $\epsilon \to 0$.

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{+\epsilon}\psi'' dx + \int_{-\epsilon}^{+\epsilon} V_0\lambda\,\delta(x)\,\psi(x)\,dx = E \cdot \underbrace{\int_{-\epsilon}^{+\epsilon}\psi\, dx}_{0} \qquad (3)$$

$$-\frac{\hbar^2}{2m}\left[\psi'(0^+) - \psi'(0^-)\right] + V_0\lambda\,\psi(0) = 0 \qquad (4)$$

Now $\psi'(0^+) = \psi_2'(0) = ik\cdot C$ Also $\psi(0) = C$

$\psi'(0^-) = \psi_1'(0) = ik - ik\cdot B$

So (4) $\Rightarrow$

$$-\frac{\hbar^2}{2m}\left(ikC - ik + ikB\right) + C\cdot V_0\lambda = 0$$

Use $C = 1+B$, elimite $B = C-1$ $\Rightarrow$

$$ikC - ik + ikC - ik - \frac{2mV_0\lambda}{\hbar^2}\cdot C = 0$$

$$\therefore\ C = \frac{ik}{ik - \frac{mV_0\lambda}{\hbar^2}} = \frac{1}{1 + i\frac{mV_0\lambda}{\hbar^2 k}}$$

$$\therefore\ T = |C|^2 = \frac{1}{1+\left(\frac{m^*V_0\lambda}{\hbar^2 k}\right)^2}$$

(d) Based on coherence and interference criteria, the first resonance happens when

$k\cdot d = 2\pi$, while energy $E = e\cdot V_1 = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \frac{\sqrt{2meV_1}}{\hbar}$

$$\therefore\ k = \frac{2\pi}{d} \Rightarrow d = \frac{2\pi}{k} = \frac{2\pi\hbar}{\sqrt{2meV_1}}\ \square$$