MS Exam ISRC Spring 2015

1: Let X_1, … X_n, be a independent sample from the Poisson density with mean λ: p(x=k; λ)= $(\lambda^k/k!)\ \varepsilon^{(-\lambda)}$: x=0,1, … ; λ>0. Give a scalar sufficient statistic for λ and show why the statistic is sufficient.

2: Suppose that we have the vector observation x related to an unknown vector parameter Θ by the equation: x=HΘ +w. w is a noise vector with: E{w}=0; E{ww'} = $\sigma^2 I$. The least square estimate of Θ is t Θ^ = $H'H)^{-1}H'x$. Show that Θ^ is the best linear unbiased estimate of Θ.

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Solution 1: (a)$L=\Pi_i\ (\lambda^{Xi}/Xi!)\ \varepsilon^{(-\lambda)}\ =\varepsilon^{(-n\lambda)}\ \lambda^{\Sigma Xi}\ /\Pi_i\ Xi!$

By the Fisher-Neyman criterion, Σ Xi is sufficient for λ.

Solution 2: Let $\Theta^{\sim}$ be any other linear estimator: $\Theta^{\sim} =((H'H)^{-1}H'+D)x$ where D≠0. Then

$E\Theta^{\sim}=E((H'H)^{-1}H'+D)(\ H\Theta +w))=(I+DH)\ \Theta \Rightarrow DH\ \Theta =0$

$E\Theta^{\sim}\Theta^{\sim'}=\sigma^2((H'H)^{-1}+(H'H)^{-1}\ (DH)'+DH(H'H)^{-1}+DD')$

$=\sigma^2((H'H)^{-1}+DD')> \sigma^2(H'H)^{-1}$