MS Exam ISRC Fall 2014

1: (a)Let X_1, … X_n, be a independent sample from the Poisson density with mean λ: $p(x=k; \lambda)= (\lambda^k/k!)\ \varepsilon^{(-\lambda)}$: $x=0,1, \dots ;\ \lambda>0$. Find the maximum likelihood estimate of λ.

(b): Is the maximum likelihood estimate unbiased? Show why.

2: Let X_1, … X_N, be a independent sample from the density: $p(x;\Theta)= 1/\Theta\ x=0<x<\Theta$, $0<\Theta$; and zero elsewhere. Let $Y_1<Y_2<Y_3<\dots<Y_N$, be the order statistics---smallest to largest. Then $p(Y; \Theta)=N!/ \Theta^N$; $0<Y_1<Y_2<Y_3<\dots<Y_N<\Theta$. Show that Y_N is a sufficient statistic for Θ.

Solution 1: (a)$L=\Pi_i\ (\lambda^{Xi}/Xi!)\ \varepsilon^{(-\lambda)} = \varepsilon^{(-n\lambda)}\ \lambda^{\Sigma Xi}\ /\Pi_i\ Xi!$

$\ln L = -n\lambda + \Sigma\, Xi\ \ln(\lambda) - \ln(\Pi_i\ Xi!)$

$(d/d\lambda)\ \ln L = -n + \Sigma\, Xi\ /\lambda = 0$

Therefore $\lambda_{ML} = (\Sigma\ X_i)\ /\ n$

$E\{\lambda_{ML}|\lambda\} = \lambda$---the estimate is unbiased.

Solution 2: $p(Y_4;\ \Theta) = \int^{yN} \ldots \int^{y3} \int^{y2} 4!/\ \Theta^N dy_1\ dy_2\ dy_3 \ldots dy_{N-1} = N\ Y_N^{N-1}/\ \Theta^N$

$p\ (Y_1,\ Y_2, \ldots,\ Y_{N-1}|\ \ Y_N;\ \Theta) = p\ (Y_1, Y_2,\ Y_3,\ \ldots Y_N;\ \Theta)/\ p(Y_N;\ \Theta)$

$=(N-1)!/\ Y_N^{N-1}$ and independent of Θ.

Y_N is a sufficient statistic