

MS Exam ISRC Fall 2012

1: Let $X_i = A \cdot f(i) + w_i$; $i=1, \dots, N$ where $\{w_i\}$ is white Gaussian noise with variance σ . $\{f_i > 0\}$ is a known sequence such that $\sum_{i=1, N} f_i \rightarrow \text{inf}$. Find the Cramer-Rao lower bound for an estimate of A. Does an efficient estimator exist? What is its variance as $N \rightarrow \text{inf}$.

Solution: $p(X, A) = C \exp\{-\frac{1}{2\sigma^2} \sum_i (X_i - A \cdot f(i))^2\}$;
 $(d \ln p / dA) = -\sigma^{-2} \sum_i (X_i - A \cdot f(i)) \cdot f(i)$
 $(d^2 \ln p / dA^2) = -\sigma^{-2} \sum_i f(i)^2$
 so $\text{Var}(\hat{A}) \geq \sigma^2 / \sum_i f(i)^2$

also

$(d \ln p / dA) = \sum_i f(i)^2 \sigma^{-2} (\sum_i (X_i \cdot f(i) / \sum_i f(i)^2) - A)$
 so $\hat{A} = \sum_i X_i \cdot f(i) / \sum_i f(i)^2$ works with variance $\rightarrow 0$