

**SOLUTION:** We have  $t_2/t_3 = 1/8$  and  $t_1/t_3 = 1/12$ . Let  $\text{sinc}(u) = \sin(u)/u$ . The input signal has period  $t_3$  and fundamental frequency  $\omega_0 = 2\pi/t_3$ . It's Fourier series is

$$\begin{aligned}
 x(t) &= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \\
 c_n &= \frac{1}{t_3} \int_{-t_3/2}^{t_3/2} x(t) e^{-jn\omega_0 t} dt \\
 &= \frac{1}{t_3} \left[ \int_{-t_2}^{t_2} A e^{-jn\omega_0 t} dt - \int_{-t_1}^{t_1} A e^{-jn\omega_0 t} dt \right] \\
 &= \frac{2A}{t_3} [t_2 \text{sinc}(n\omega_0 t_2) - t_1 \text{sinc}(n\omega_0 t_1)] \\
 &= \frac{2A}{t_3} [t_2 \text{sinc}(2\pi n t_2 / t_3) - t_1 \text{sinc}(2\pi n t_1 / t_3)] \\
 &= \frac{A}{12} [3 \text{sinc}(\pi n / 4) - 2 \text{sinc}(\pi n / 6)] \\
 c_0 &= A/12 \\
 c_1 = c_{-1} &= \frac{A}{12} \left( \frac{3\sqrt{2}/2}{\pi/4} - \frac{2(1/2)}{\pi/6} \right) = \frac{A}{2\pi} (\sqrt{2} - 1)
 \end{aligned}$$

Note that  $h(t) = (B/t) \sin(1.5\omega_0 t)$ . The LTI system is a low pass filter with frequency response

$$H(\omega) = \begin{cases} B\pi & \text{if } |\omega| < 1.5\omega_0 \\ 0 & \text{else} \end{cases}$$

so in the Fourier series for  $x(t)$ , only the terms corresponding to  $n = -1, 0, 1$  pass through the filter. Thus,

$$\begin{aligned}
 y(t) &= B\pi \sum_{n=-1}^1 c_n e^{jn\omega_0 t} \\
 &= B\pi (c_0 + c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t}) \\
 &= B\pi (c_0 + 2c_1 \cos(\omega_0 t)) \\
 &= AB \left( \frac{\pi}{12} + (\sqrt{2} - 1) \cos(2\pi t / t_3) \right)
 \end{aligned}$$