

Math Question (equal weight each part)

Part 1

(i) Find the eigenvalues and eigenvectors of the following matrix

$$M = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$$

(ii) Using a linear combination of the eigenvectors, construct the basis vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(iii) If one number is changed in the matrix, then the corresponding eigenvectors will be orthogonal. Determine this number and verify that the resulting eigenvectors are orthogonal.

Part 2

A joint probability distribution $p_{XY}(x, y)$ is given by

$$p_{XY}(x, y) = A(8x + 14yx - 21y - 12)$$

where $0 < x < 1/2$, $-1 < y < -1/2$, and A is a constant.

i) Find A

ii) Find the marginal distributions for $p_X(x)$ and $p_Y(y)$

iii) Are the random variables defined by X and Y independent? (Justify your answer.)