

Math Question (equal weight each part)

Part 1

You are given the following vector field in rectangular coordinates

$$\mathbf{F} = (y \cos x + y^2)\hat{\mathbf{x}} + (\sin x + 2xy - 2y)\hat{\mathbf{y}}$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ are unit vectors.

- (i) Prove that this field is conservative.
- (ii) Determine the potential function $\phi(x, y)$ of this field such that $\mathbf{F} = \nabla\phi$.

Part 2

(i) A circuit contains five components and will fail if two out of the five components fail. If the failure of each component is random and the probability of failure of each component is the same, then what is the maximum failure rate for each component if the failure rate of the overall circuit is to be less than 10%?

(ii) Can the eigenvectors of the following matrix

$$\begin{bmatrix} 1 & 2 \\ 7 & 6 \end{bmatrix}$$

be used as an orthogonal set of basis vectors? Justify your answer.