Math Question

(You can attempt all three parts. A passing grade is at least 50% for two out of the three parts.)

Part 1

You are given the following differential equation

$$\frac{d^2y}{dt^2} + b\frac{dy}{dt} + 16y = 0$$

where b is a real constant. Determine the value(s) of b for which:

1. The system is under damped.
2. The system is critically damped.
3. The system is over damped.
4. The system is underdamped with the decay rate equal to the oscillation frequency.

Part 2

1. You are given the following matrix

$$M = \begin{bmatrix} 1 & 2+j \\ c & d \end{bmatrix}.$$

   Determine the allowing value(s) of c and d such that M is a hermitian matrix. (Note $j = \sqrt{-1}$.)

2. A second matrix is given by

$$M = \begin{bmatrix} 1 & a \\ 2 & 1 \end{bmatrix}.$$

   (a) Determine the eigenvalues of M.
   (b) Let the larger eigenvalue be x and the smaller eigenvalue be y. Determine the value of a such $x/y = -3$.

Part 3

A joint probability density function in polar coordinates (r, ϕ) is given by

$$f_{\mathbf{r}\boldsymbol{\phi}}(r, \phi) = \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

1. Determine the marginal probability density function $f_{\boldsymbol{\phi}}(\phi)$.
2. Determine the probability density function $f_{\boldsymbol{xy}}(x, y)$ in retangular coordinates.
3. Are the random variables $\mathbf{x}$ and $\mathbf{y}$ independent? Justify your answer.