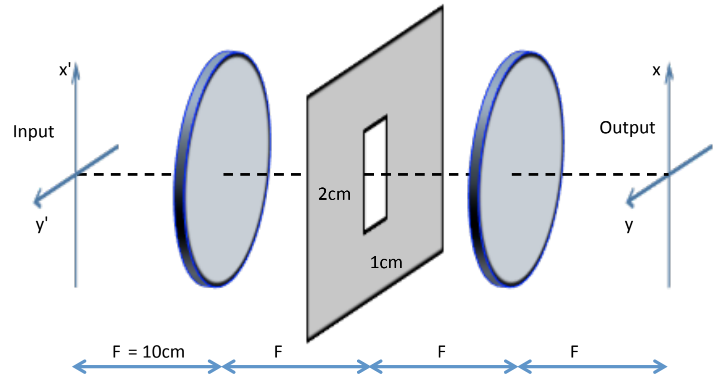
Photonics MS Exam Fall 2015 Solutions

Question 1: Imaging & Diffraction



Part 1: A 4-F relay imaging system is made of two ideal 10cm focal length lenses, and 2 by 1 cm rectangular aperture, arranged as shown at right. The aperture is 100% transmissive within the rectangle, and opaque elsewhere. The input light is 500nm in wavelength. What is the functional dependence **of the impulse response**, and the **full widths δx and δy** (between zero intensity) of the spot?

The impulse response is proportional to the inverse Fourier transform of the pupil function p(x.y), evaluated at $f_x = x/\lambda F$, $f_y = y/\lambda F$.

$$P(f_x, f_y) = \mathcal{F}^{-1}\{ p(x,y) \} = \mathcal{F}^{-1}\{ \text{rect}(x/W_x)\ \text{rect}(y/W_y) \} = W_x \,\text{sinc}(W_x f_x)\, W_y \,\text{sinc}(W_y f_y)$$

where $W_x = 2\text{cm}$ and $W_y = 1\text{cm}$

$\rightarrow h(x,y)\ \alpha\ \text{sinc}(W_x x/\lambda F) \times \text{sinc}(W_y y/\lambda F) = \underline{\text{sinc}(x/2.5\mu\text{m}) \times \text{sinc}(y/5\mu\text{m})}.$

The sinc function has zero amplitude when the argument is ±1, so the full width of the spots formed

$\underline{\delta x = 2\times 2.5\mu\text{m} = 5\mu\text{m}}$, and $\underline{\delta y = 2\times 5\mu\text{m} = 10\mu\text{m}}$.

Part 2: A spatial filter has sinusoidally varying amplitude transmission ranging from zero to 1, with a spatial period of 10 microns. **Express t(x) and provide the angular spectral decomposition** (angular spectra) of the transmitted wavefront when this filter is illuminated by an on-axis plane wave of wavelength 500nm.

The sinusoidal transmission function (offset vertically to stay positive, and normalized) is

$$t(x,y) = t(x) = \tfrac{1}{2} [1 + \cos(2\pi x/T)] = \tfrac{1}{2} e^{j0} + \tfrac{1}{4} e^{j2\pi x/T} + \tfrac{1}{4} e^{-j2\pi x/T}$$

This is a sum of three harmonics, so we identify the zeroth order with $k_x = k_y = 0$ and amplitude weight ½, and first and minus first orders with $k_y = 0$ and $k_x = \pm 2\pi/10\mu\text{m}$ and amplitude weight ¼.

This is sufficient, but you can express the transmitted wavefront as a sum of three paraxial plane waves:

$$U(x,y,z) = \tfrac{1}{2} e^{-j2\pi z/\lambda} + \tfrac{1}{4} e^{-j(2\pi x/T + 2\pi z/\lambda)} + \tfrac{1}{4} e^{-j(-2\pi x/T + 2\pi z/\lambda)}$$

Part 3: This spatial filter is placed in contact with the rectangular aperture in the system above. Without deriving an explicit expression for the overall system output, **calculate the spatial separation** between the outputs generated by the **minus first and first output orders** of the filter.

This question can be solved by converting the wavefront tilt in the Fourier plane to lateral shift in the output image plane, or you could consider the lateral offset of the projected aperture as it propagates between the pupil and second lens in the (telecentric) imaging system.

The angle between first and minus first orders is twice the angle between the first and zeroth order,

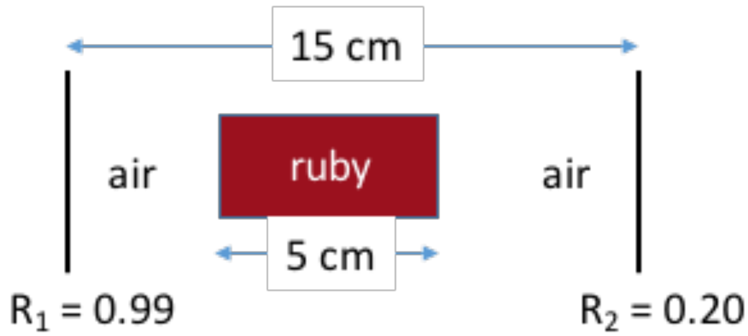
$$\theta = \sin^{-1}(k_x/k) = \sin^{-1}((2\pi/10\mu\text{m})/ 2\pi/0.5\mu\text{m})) = \sin^{-1}(0.5/10) = \sin^{-1}(0.05) \cong 0.05 \text{ rad}$$

The lateral offset is then

$$\Delta x = 2 \times F \tan(\theta) = 2 \times 10\text{cm x} \tan(0.05\text{rad}) \cong 20\text{cm} \times 0.05 = 1\text{cm}.$$

Question 2: Lasers

A crystal of ruby of length 5 cm and cross-sectional area 1 cm^2 is placed as shown between a highly-reflecting mirror (intensity reflection coefficient 0.99) and a weakly-reflecting mirror (intensity reflection coefficient 0.20). The distance between the mirrors is 15 cm.



Properties of Ruby: absorption coefficient is 0.5 /cm, refractive index is 1.8 at wavelength of 700 nm and when it is optically pumped, the spontaneous emission cross-section is 2.5 x 10^(-20) /cm^2.

a) What is the gain (units: /cm) needed to achieve lasing threshold? [2 points: 1 point for correct equation, 1 point for correct numerical value]

Reflection coefficient at the ruby-air interface R = (1.8-1)/(1.8+1) = 8.1%
Therefore, transmission coefficient T = 1- R0 = 92%
For simplicity, OK to ignore Fabry-Perot effects between air-Ruby interfaces.

Round trip loss x gain = 1
R1 R2 T^4 exp(-2 a L) exp(2 g L) = 1
where a = 0.5 / cm, L = 5 cm

Answer: g = 0.7 /cm.

b) How many Ruby atoms are population-inverted at lasing threshold? [3 points: 2 points for correct equation(s), 1 point for correct numerical value]

Threshold inversion (density) ΔN = g / s = 2.8 x 10^(19) /cm^3.
Number of inverted atoms N_inv = ΔN x C.S. Area x L = 1.4 x 10^(20).

c) What is the photon lifetime in the passive cavity? [3 points: 2points for correct equation, 1 point for correct numerical solution]

Round-trip time:
t_rt = (2*L_air + 2*L_ruby*n_ruby)/c
Cavity photon lifetime:
1/t0 = [1 – (R1 R2 T^4) exp(-2 a L)] / t_rt

So, t0 = 1.28 ns

(Note: (R1 R2 T^4) exp(-2 a L) is a pretty small number, about 0.001 – and can be ignored for these numerical values. So, this is an easy question – a simple distance / speed guesstimate will work.)

d) Assume that each of the population-inverted atoms emits a photon at 700 nm with uniform probability within the photon lifetime (calculated in part c). What is the output power of this laser? [2 points: 1 points for correct equation, 1 point for correct numerical value]

Since the left mirror is much more highly reflective than the right one, we can assume that the emitted photons emerge only from one side.

In the steady state, the emitted power accounts for the photons emitted by the atoms, so

P = (h c / lambda) x N_inv / t0 = 624 MW

(Note, this is a back-of-the-envelope of a Q-switched pulse, in the "flat-top" approximation. Typical peak powers are indeed a few hundred MW.

These numbers evaluate to about 8 J/cm^2 which is typical for a laser, e.g., used in tattoo removal.
https://www.astanzalaser.com/our-lasers/eternity/ 2 – 14 J/cm^2.)