

## Math Question

(You can attempt all three parts. A passing grade is at least 50% for two out of the three parts. )

### Part 1

Find the solution of the initial value problem

$$\frac{dy}{dt} - 2ty = 1 \quad y(0) = 1$$

Note:  $\int_0^t e^{-\tau^2} d\tau = (\sqrt{\pi}/2)\text{erf}(t)$  where  $\text{erf}(t)$  is the error function.

### Part 2

Find the general solution of the following matrix equation

$$\begin{bmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}$$

### Part 3

A uniform random variable  $X$  is defined over the interval  $[0, 2]$ . A second independent random variable  $Y$  has a probability density function  $P_Y(y) = 2e^{-2y}u(y)$  where  $u(y)$  is a unit-step function. Find and sketch the probability density function  $P_Z(z)$  for the random variable  $Z = X + Y$ .