Sample Questions

(1) Phosphate buffered saline (PBS) is a buffer solution commonly used in biological research. It is a water-based salt solution containing sodium phosphate, sodium chloride and, in some formulations, potassium chloride and potassium phosphate. The osmolarity and ion concentrations of the solutions match those of the human body (isotonic).

The most common composition of PBS (1X) has the following components:

Salt	Concentration (mmol/L)
NaCl	137
KCl	2.7
Na2HPO4	10
KH2PO4	1.8

The above buffer solution may give the approximate ionic concentration of 10 mM HPO_4^{-2}, 157 mM Na^+, and 139.7mM Cl^-.

(a) Calculate the Debye length for 1X (undiluted) PBS and PBS that is 10 times diluted by deionized water.

(b) Assuming a quantum dot (10nm diameter) with a surface charge density of -10^{-7} C/cm^2 is in the 1X PBS buffer, find the potential at the surface of the particle and at 1nm from the surface of the gold nanoparticle.

(c) Answer (b) assuming the same quantum dot is in 10 times diluted PBS.

(d) Discuss in which buffer (1X PBS or 10-times diluted PBS) there is a greater chance for quantum dots to form aggregates.

(2) A liposome consists of a 8nm lipid bilayer membrane (ε=3.5) shell and a 50 nm diameter liquid core (ε=78). The electrolyte inside the liquid core is the same as the outside medium except that the core contains non-zero net charge (i.e. not charge neutral).

(a) Calculate the capacitance of the liposome.

(b) Calculate the total amount of charge of the liposome if the membrane potential is -30mV.

(c) Calculate the E-field in the membrane.

(3) In this problem we will find the electrophoretic mobility for biological cells and molecules. Assume ε=78, viscosity η=0.9 cP (1cP=10^{-3} dyne-s/cm^2)

(a) Write down the expression of electrophoretic mobility of a particle in fluid.

(b) Calculate the mobility for a spherical bacteria cell that has 1um diameter. The zeta potential of the bacteria is -50mV.

(c) The membrane thickness of the bacteria is 4nm, calculate the total charge within the cell. (You can ignore any cell wall and the fine structures between the cell wall and the membrane)

(d) Calculate the diffusivity of the bacteria.

(4) Do the same problem as (3) with a mammalian cell (e.g. red blood cell (RBC))
   (a) Write down the expression of electrophoretic mobility of a particle in fluid.
   (b) Calculate the mobility for a RBC that can be modeled as a 7um diameter disk with 2um thickness. The zeta potential of RBC is -15mV.
   (c) The membrane potential and membrane thickness of RBC is -10mV and 10nm, respectively. Estimate the total amount of charge within a RBC.
   (d) Calculate the diffusivity of RBC.

(5) Assume that 10^3 potassium ion channels open at the same time and each ion channel let 10^8 K+ ions pass through per second following the concentration gradient.
   (a) Calculate the number of K^+ leaving the cell over a period of 10ms and the magnitude of ion current.
   (b) Estimate the ion travel speed over the ion channel assuming the ion channel is 10nm long.
   (c) Assume the cell has a volume of 1pL (10^{-12} liter) and the initial K+ concentration is 100 mM. Calculate the change of K^+ concentration within the cell after 10 ms assuming that the potassium concentration is uniform everywhere inside the cell.
   (d) Assume that all 10^3 opened K^+ ion channels are located over a very small area of the cell membrane and the K^+ diffusivity is $1x10^{-5}$ cm^2/sec. Estimate the change of *local* K^+ concentration. This gives a more realistic result than (c).

(6) Na+/K+-ATPase (sodium-potassium adenosine triphosphatase) is a Na+/K+ ion pump found in all animal cells. The pump moves 3 sodium ions out and 2 potassium ions in each time. It helps maintain resting potential, avail transport, and regulate cellular volume. It also functions as signal transducer/integrator to regulate MAPK pathway, ROS, as well as intracellular calcium. However, 30-50% of total energy a cell consumes is to maintain the function of this very important ion pump.
   (a) The ATP hydrolysis reaction to produce ADP releases 7.3Kcal/mole. Calculate the amount of energy in electron volt (eV) released by each ATP molecule.
   (b) Assuming that the Na+/K+ ion pump works against a 10:1 concentration gradient for both Na^+ and K^+ and the cell has a membrane potential of -70mV, calculate the energy needed for the pump to remove 3 Na^+ from the cell and bring in 2 K^+ into the cell against the 10:1 concentration gradient for both ions. Write your answer in electron volt (eV).
   (c) Calculate the number of ATP needed to support the function of a Na+/K+ ion pump to transport 3 Na^+ and 2 K^+ assuming 100% energy transfer efficiency.
   (d) Assume the Na+/K+ ion pump can pump 1000 Na^+ ions per second, calculate the power consumption (in Watts) of each ion pump.

(7) This problem is about osmotic pressure. Assume two chambers are separated by a water permeable membrane. Chamber A contains water only and chamber B contains water and 1 mM of glucose which cannot permeate through the membrane.

(a) Calculate the osmotic pressure at 37C. Which chamber will have a greater pressure? Represent the osmotic pressure in Pascal and in mmHg.
(b) Besides glucose, we now dissolve 5mM NaCl in chamber A and 1 mM NaCl in chamber B. Assume neither Na^+ nor Cl^- can permeate through the membrane. Calculate the pressure difference between the two chambers again. Which chamber will have a greater pressure?
(c) Keeping the initial contents of chamber A and chamber B the same as (b), but we use a membrane that contains Na+ ion channels to separate the two chambers. Now the membrane is not only water permeable but also Na^+ permeable. The permeability ratio for H_2O and Na^+ is assumed to be 2:1. Calculate the pressure difference between the two chambers and indicate which chamber has greater pressure.

(8) We analyze blood flow in blood vessel in a simplified model. Assume blood flows in a 5mm inner diameter tube. The viscosity of the fluid is 5 cP (i.e. 0.05 dyne.s/cm^2). Assume that a laminar flow is established under a pressure gradient 10 mmHg/cm. Calculate
(a) The flux in mL/min,
(b) The amount of shear stress on the wall of the tube,
(c) The power consumption for the flow traveling through a 1m long tube.

Provided the tube is obstructed and the inner diameter is reduced to 3mm, repeat (a), (b), (c).

(9) Assume a vesicle has a lipid bilayer membrane and a diameter of 1um. Inside the diameter the concentration of KCl is 100 mM. The surrounding medium has a KCl concentration of 20mM. Assume that there exist K^+ channels but no Cl^- channels in the lipid membrane, calculate
(a) the membrane potential,
(b) the final K^+ and Cl^- concentration after equilibrium is reached.
(c) If the K^+ flux through the K+ channels is 10^{10}/s and the cell can be modeled as a 10um diameter sphere, estimate approximately how long it takes for the system to reach equilibrium.

(10) If we model a double strain DNA as a rod of 2nm cross section with a charge density of $-2x10^{-5}$ (C/cm^2), calculate
(a) The potential profile if the DNA is in a salt solution with a NaCl concentration of 10 mM.
(b) Na^+ and Cl^- concentration right next to the DNA molecule.
(c) The concentration of Ca^{+2} if we add 1uM of C_aCl_2 in the solution. Also comment how the concentration profile of bivalent ions is different than that of monovalent ions.
(d) Describe how the ionic strength may affect the hybridization and stability of dsDNA. What will happen if the salt concentration is too low (e.g. deionized water)? What will happen if the salt concentration is too high?

(11) The PH dependence of the amount of charge of a protein molecule can be written approximately as Zp=5(PH-5.0). The protein, at a concentration of 1uM, is inside a 1um diameter vesicle with 8nm lipid membrane thickness. The initial PH value of the vesicle is PH=4.0 due to the HCl concentration of 0.1mM. The PH value of the outside medium is

PH=7.0. Assume no other ions or molecules except protons can pass through the membrane, calculate

(a) The initial membrane potential.

(b) The membrane potential after the system reaches equilibrium.

(c) The final PH value when the system reaches equilibrium.