

# Math Question

## Problem 1

A vector field is given by

$$\mathbf{V}(x, y, z) = e^x \sin y \hat{\mathbf{i}} + e^x \cos y \hat{\mathbf{j}} + 5 \hat{\mathbf{k}}$$

where  $\hat{\mathbf{i}}$ ,  $\hat{\mathbf{j}}$  and  $\hat{\mathbf{k}}$  are unit vectors in the  $x$ ,  $y$  and  $z$  directions respectively. Determine the work done by the vector field on an object moving along a smooth curve from the point  $(-\pi, 0, \pi)$  to the point  $(\pi, 0, -\pi)$  and then back to the original point. (No credit without justification.)

## Problem 2

Find the eigenvectors and eigenvalues of the following matrix

$$\mathbb{M} = \begin{bmatrix} 3 & 10 \\ 2 & 4 \end{bmatrix}$$

- Are the eigenvectors orthogonal? (No credit without justification.)
- Express the vector

$$\mathbf{a} = \begin{bmatrix} -14 \\ 11 \end{bmatrix}$$

as a linear combination of the eigenvectors and using this decomposition, express the vector matrix product

$$\mathbf{b} = \mathbb{M}\mathbf{a}$$

as a linear combination of the eigenvectors.