Suppose a discrete linear time-invariant system has output $-\delta[n] + 2\delta[n-2]$ when the input is an impulse $\delta[n-1]$ centered at $n = 1$. What is the input to the system when the output is $-2\delta[n+3] + \delta[n+2] + 4\delta[n+1] - 4\delta[n] + 4\delta[n-2]$?

**SOLUTION:**

\[
y[n] = -2\delta[n+3] + \delta[n+2] + 4\delta[n+1] - 4\delta[n] + 4\delta[n-2] \\
h[n] = -\delta[n+1] + 2\delta[n-1] \\
Y(z) = -2z^3 + z^2 + 4z - 4 + 4z^{-2} \\
H(z) = -z + 2z^{-1} \\
X(z) = \frac{Y(z)}{H(z)} = \frac{-2z^3 + z^2 + 4z - 4 + 4z^{-2}}{-z + 2z^{-1}} \\
= z^{-1} \cdot \frac{2z^5 - z^4 - 4z^3 + 4z^2 - 4}{z^2 - 2} \\
= 2z^2 - z + 2z^{-1} \\
\therefore x[n] = 2\delta[n+2] - \delta[n+1] + 2\delta[n-1] \]