Suppose we take $N$ independent samples, $X_i$, with the exponential density:

$$p(X_i) = \lambda \exp\{-\lambda X_i\} \text{ for } X_i > 0 \text{ and } 0 \text{ otherwise.}$$

We wish to estimate $\lambda$. What is a minimal dimension sufficient statistic for $\lambda$? Call this statistic $\Phi$. Give an unbiased estimate of $\lambda$ in terms of $\Phi$: $\lambda^\wedge = f(\Phi)$. How does the variance of $\lambda^\wedge$ compare with any other unbiased estimate of $\lambda$. Are there qualifications regarding the last result.