Instructions:

All work to be done on the attached sheets. Write your name at the top of every sheet.

Problem 1. From gate level circuit shown below, derive the optimized circuit with minimum number of gates. (15 points)
Problem 2. Let $F(A,B,C,D) = \sum m(1,3,4,5,7,9,11)$, with don’t cares $d(A,B,C,D) = \sum m(6,8,10)$

(a) Draw the K-Map for $F$. (10 points)

(b) List all the prime implicants, and the essential prime implicants. (10 points)

(c) Derive the optimized function $F$. How many literals does it have? (5 points)
**Problem 3.** Consider a state diagram consisting of 6 states.  

(a) How many flip-flops are needed if minimal encoding is used for the states in the state diagram?

(b) How many flip-flops are needed if one-hot encoding is used for the states in the state diagram?

(c) Let S be a sequential circuit derived using minimal encoding. How many valid and invalid (don’t care) states will be present in the state table (or state diagram) extracted from the sequential circuit S?

(d) Let R be a sequential circuit derived using one-hot encoding. How many valid and invalid (don’t care) states will be present in the state table (or state diagram) extracted from the sequential circuit R?
**Problem 4.** A sequential circuit has two inputs and two outputs. The inputs \((X1, X2)\) represent a 2-bit binary number, \(N\). If the present value of \(N\) is greater than the previous value, then \(Z1=1\). If the present value of \(N\) is less than the previous value, then \(Z2=1\). Otherwise, \(Z1\) and \(Z2\) are 0.

Derive a Mealy machine state diagram. (*Hint:* The machine needs only five states.) (20 Points)

Sample inputs and outputs:
\[
\begin{align*}
X1 & : 0011001 \\
X2 & : 1001110 \\
Z1 & : 0011001 \\
Z2 & : 0100100
\end{align*}
\]