Electromagnetism (ECE 107), MS Comprehensive Exam, Spring 2012

A plane wave is incident from the left with an incident angle $\theta_i$ on the interface (at $z = 0$) between two half-spaces. The left half space is filled with a medium with $\epsilon_1, \mu_0$. The electric field of the incident wave is given by

$$E^i = H_0 \left( \hat{x} \cos \theta_i - \hat{z} \sin \theta_i + \hat{y}/\sqrt{3} \right) e^{-j k_0 (x \sin \theta_i + z \cos \theta_i)},$$

such that the electric field vector of the incident wave has a $\pi/6$ angle with respect to the $xz$ plane. Here, $\hat{x} \cos \theta_i - \hat{z} \sin \theta_i$ and $\hat{y}$ field components correspond to the parallel and perpendicular polarizations, respectively.

a) The right half space is filled with a dielectric medium having $\epsilon_2, \mu_0$ with $\epsilon_1 > \epsilon_2$ (see Fig. a). Write an expression for the reflection coefficient and reflected electric field. Give expressions for the critical angle and Brewster angle.

b) Referring to item a, i. For the case $\theta_i < \theta_c$ find the condition on $\theta_i$ under which the electric field vector of the reflected wave has an angle of $\pi/3$ with respect to the $xz$ plane. Only set up the conditions without solving for the required $\theta_i$.

ii. For the case $\theta_i > \theta_c$ find the condition on $\theta_i$ under which the phase shift between the parallel and perpendicular reflected wave components is $\pi/3$. Only set up the conditions without solving for the required $\theta_i$.

c) Now, the right half space is filled with a perfect electric conductor (see Fig. b). Give the reflection coefficient and write an expression for the reflected electric field. Using the boundary conditions, give the surface current $J_s$ on the conductor (at $z = 0$).