Solution

a. First, recognize that

\[ f_T = \frac{g_m}{C_{gs}}. \] (5)

Next, we start with the long channel drain current equation.

\[ I_d = \frac{\mu C_{ox} W}{2L} \left( V_{gs} - V_{TN} \right)^2 \]

We find the gm by taking the derivative with respect to Vgs.

\[ g_m = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{TN} \right). \] (5)

Next, we need to find the Cgs in saturation.

\[ C_{gs} = \frac{2}{3} C_{ox} W L. \] (5)

Therefore, we can find the fT.

\[ f_T = \frac{\mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{TN} \right)}{\frac{2}{3} C_{ox} W L} = \frac{3 \mu_n}{2L^2} \left( V_{gs} - V_{TN} \right) = 75 \text{ GHz} \] (5)

b. Series feedback at the source will not change the fT since the transconductance is reduced by an equal amount to the increase in the impedance. Note that the transconductance is reduced

\[ g_m^{' \prime} = \frac{g_m}{1 + g_m s L} \]

Suggests that the source inductor acts as degeneration which reduces the output current. However, the input impedance becomes

\[ Z_m = sL + \frac{1}{sC_{gs}} + \frac{g_m s L}{C_{gs}} \]

If you assume that L is used to resonate out Cgs, then

\[ |h_{21}| = \left| \frac{g_m L}{C_{gs}} \right| \frac{g_m}{1 + g_m s L} = 1 \]

\[ \left( \frac{g_m}{C_{gs}} \right)^2 \frac{C_{gs} L}{1 + g_m \omega_L \omega_L} \approx \left( \frac{g_m}{C_{gs}} \right)^2 \frac{1}{\omega_L} = 1 \]

\[ \omega_L = \frac{g_m}{C_{gs}} \]

Then, the fT remains unchanged. 15 pts for the mathematical argument.

c. The voltage across the input is determined by the input impedance.
The input impedance is maximum when the inductance resonates with $C_{gs}$. Therefore, the voltage across the gate source capacitor is

$$V_{gs} = i_s R_p = i_s Q \omega_L L$$

$$i_s = g_m V_{gs} = g_m Q \omega_L L i_s$$

Since $LC_{gs} = \omega_0^2$, we find

$$h_{21} = \left| \frac{i_s}{i_s} \right| = g_m Q \omega_L \frac{1}{C_{gs} \omega_0^2} = Q$$

Therefore the $f_T$ should be $Q$ times larger (500 GHz). There are caveats to this – particularly about the $Q$ of the inductor remaining high over all frequency. A more exact answer is found from

$$h_{21} = \frac{\omega_L}{\omega_0} \frac{j \left( \frac{\omega}{\omega_0} \right)}{1 - \left( \frac{\omega}{\omega_0} \right)^2 + j \frac{1}{Q} \left( \frac{\omega}{\omega_0} \right)}$$

If one differentiates with respect $\omega_0$, you will find that the peak $f_T$ is $Q$ times higher.

6 pts for determining the observing the change in the input current/output current relationship. 3 pts for the correct evaluation of $f_T$. 