1) Derive an expression for the Noise Figure of the following circuit:

Assume all of the capacitances in the transistor are zero. Assume the transistor is noiseless and $r_{ds} = \infty$. 
2) A transistor has the following relationship:

\[(i_d + I_D) = \alpha_1 (V_{gs} + V_{GS}) + \alpha_2 (V_{gs} + V_{GS})^2\]

Derive an expression for the nonlinear small-signal input voltage, i.e.

\[i_d = \beta_1 V_{in} + \beta_2 V_{in}^2\]

You only need the first two \(\beta\) terms.

There are \(\text{dc}\) bias terms \(V_G\) and \(I_D\) and small-signal terms \(V_{in}\) and \(i_d\).
3a) Design an LC matching network at 2 GHz for a load with an impedance of 20-j80 Ω. What is the reflection coefficient at 1.6 GHz and 2.5 GHz? Do it by hand using the Smith Chart. Use ideal L and C. Note that at 1.6 GHz and at 2.5 GHz, the capacitive component of the load has changed linearly with frequency.

b) Consider the 4-port network shown below. Note that Ports 3 and 4 are not grounded, but independent ports, just like Ports 1 and 2. The line impedance is $Z_0$.

![Diagram](image.png)

Calculate the scattering matrix. Use symmetry and reciprocity to greatly simplify your calculations. What is the power dissipated in R (when all ports are matched)? You can do the power loss using the S-matrix and not circuit analysis.