A small piece of graphene sheet with dimension \( L \times W \) is connected to source and drain electrodes on both ends. It is sitting on top of a gate electrode with a thin layer of silicon oxide (\( \varepsilon = 3.9 \)) in between with thickness \( t_{\text{oxide}} \). Electron rest mass is \( m_0 = 9.11 \times 10^{-31} \text{kg} \).

1. Given graphene band structure \( E(k) = \hbar v_F |k - k_0| \), calculate the 2D electron density \( n_{2D} \) in the graphene, if \( E_F \) is 0.1 eV above the Dirac point. \( k_0 \) is one of the six corners of the 1st Brillouin zone. Use \( v_F = 1 \times 10^6 \text{ m/s} \).

2. Following #1, given \( L = 1 \mu \text{m}, W = 100 \text{ nm} \) and if the mobility of this piece of graphene is 2,000 \( \text{cm}^2/\text{s} \), what is the resistance \( R \) measured between the source and drain?

3. If we can shrink the graphene width \( w \) and make it into a narrow strip, calculate how narrow should it be to have only a single subband occupied given the \( E_F \) level in #1. Use particle-in-a-box model assuming infinite high barriers outside the graphene edges and ignore edge chirality effects.

4. What is the resistance \( R \) at #3’s condition?

5. If both contacts are bad, thus forming barriers so that Coulomb blockade occurs, sketch in a diagram what would the current vs. gate voltage data look like. What is its periodicity if \( t = 5 \text{ nm}, L = 500 \text{ nm}, W = 50 \text{ nm} \)? What is the requirement on temperature for this to be observable?

6. Briefly discuss what might happen to the graphene band structure and its conductance if the sample is a bilayer (double-layer) graphene.