1. Let $X(t)$ be a random telegraph signal

$$X(t) = X(0) (-1)^{N(t)}$$

where $N(t)$ is a classical Poisson process (independent increments, constant rate $\lambda$, $N(0) = 0$). For $t \geq s$

$$P(N(t) - N(s) = m) = \frac{\lambda(t-s)^m}{m!} e^{-\lambda(t-s)}, \ m = 0, 1, 2 \ldots$$

and the random variable $X(0)$ is independent of $N(t)$ with

$$P[X(0) = 1] = P[X(0) = -1] = 1/2.$$  

It is desired to obtain an estimate of $X(t)$ based on a single earlier observation using a mean square error criterion

$$\mathcal{E} = E[(X(t) - \alpha X(t-t_o)^2), t_o > 0].$$

(A) Determine the value of $\alpha$ that minimizes $\mathcal{E}$.

(B) Find the largest value of $t_o$ for which $\mathcal{E}_{\text{min}} \leq 2/3$. 