The joint mass function for \((n_1, \ldots, n_R)\) is given by \(C \prod p_i^{n_i}\)

The log likelihood is
\[
I(p;n) = \sum n_i \log(p_i) + C.
\]

differentiating \(\nabla I(p;n) = (n_1 p_1^{-1}, \ldots, n_R p_R^{-1})\)

Solving \(\hat{p}_i = n_i/N; i = 1 \text{ or } 2\)

Since \(E\{ \hat{p}_i \mid \{X_i\} \} = p_i\), the estimate is unbiased.

\((n_1, \ldots, n_R)\) is a sufficient statistic and the ML estimate is unbiased. I will assume the statistic is complete. Thus \((n_1/N, n_2/N)\) is the MVUE of \(p_1\) and \(p_2\) is minimum variance.