4. (a) Let \( X(t) \) be the number of calls till time \( t \). Then, \( X(t) \sim \text{Poisson}(\lambda t) \). Hence, the probability that fewer than two calls arrive in the first hour is

\[
P(X(1) = 0) + P(X(1) = 1) = e^{-\lambda} + \lambda e^{-\lambda} = e^{-\lambda}(1 + \lambda),
\]

where \( \lambda = 4 \) as given in the problem.

(b) From memoryless property of exponential random variables and independent increments,

\[
P(X(2) \geq 8 \mid X(1) = 6) = P(X(1) \geq 2) = 1 - (1 + \lambda)e^{-\lambda}.
\]

(c) Let \( Y_i, i \geq 1 \), be the interarrival time between \((i - 1)^{st}\) and \(i^{th}\) calls. Then, \( Y_i \sim \text{exp}(\lambda) \). If \( X \) is the amount of time the person waits, then \( X = \sum_{i=1}^{15} Y_i \), and

\[
E[X] = E[\sum_{i=1}^{15} Y_i] = \sum_{i=1}^{15} E[Y_i] = 15 \frac{1}{\lambda} = \frac{15}{4}.
\]

(d) Given that there were exactly eight calls, the arrival time of each call is uniformly distributed over \([0, 2]\) and these are independent. Hence, the probability that exactly five of them arrived in the first hour is

\[
\binom{8}{5} \frac{1}{2^8} = \frac{8!}{5! \cdot 3! \cdot 2^8}.
\]

Alternately, one can derive this result by computing

\[
P(X(1) = 5 \mid X(2) = 8) = \frac{P(X(1) = 5, X(2) - X(1) = 3 \mid X(2) = 8)}{P(X(2) = 8)} = \frac{\lambda^5 e^{-\lambda} \cdot \lambda^3 e^{-\lambda}}{(2\lambda)^8 e^{-2\lambda}} = \frac{8!}{5! \cdot 3! \cdot 2^8}.
\]

(c) Again, given that exactly \( k \) calls arrived in the first four hours, the arrival time of each call is \( \text{Unif}(0, 4) \) and these are independent. Hence, the probability that exactly \( j \) of them arrived in the first hour is \( \frac{k!}{j!(k-j)!} \left( \frac{1}{4} \right)^j \left( \frac{3}{4} \right)^{k-j}, 0 \leq j \leq k \).